The Logistic Equation

The weather is chaotic. This means that unless we know perfectly what the weather is doing at a given time (down to the scale of mm), we can not make a perfect weather forecast. Small differences in the current conditions can lead to big differences in how the weather develops. There are other, simpler situations where a small change in the starting conditions can lead to a big change in the result.

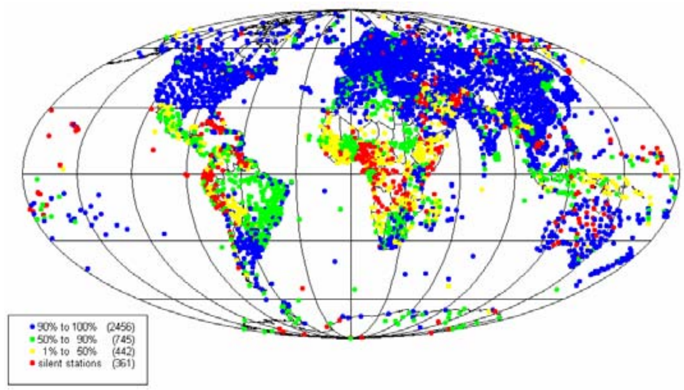

Figure 1. The global network of WWW stations colour coded to show reporting rates. Data sparse areas and low reporting rates are clearly visible. :Source, WMO (2003) Twenty-First Status Report on Implementation of the World Weather Watch: Forty years of World Weather Watch, WMO No. 957, p49.

Where are most weather observations made? Where is the weather observed infrequently?

Consider a quantitity, x, whose current value is known. If we also know how it evolves in time, we can say what the value of x will be a short time Δt later. If we call the current time t_1, then we could call $t+\Delta t=t_2$, $t_2+\Delta t=t_3$ etc. In other words, we can use the value of x at any time t, x_t, to calculate its value Δt later, x_{t+1}.
The Logistic equation is one particular way x could evolve.

It is written $$x_{t+1} = kx_t(1-x_t)$$

This means each subsequent value of x depends on the previous value.

Open the Excel file ‘Logistic_equation.xls’.

In the spreadsheet, the values of k and $x_{t=0}$ are set at the top.

The table allows you to see the value of x over 60 iterations

How does *x* change with time?

Try values of k between 2.4 and 3.5.

What do you notice happening?

At what value of k does the behaviour of the system go from being 'normal' to being 'chaotic'?

Choose a value of k in the normal region, and vary $x_{t=0}$ slightly (e.g.from 0.2 to 0.21)

How does *x* change with time?

Choose a value of k in the chaotic region, and vary $x_{t=0}$ slightly (e.g.from 0.2 to 0.21)

How does *x* change with time?

Chaos is when a small change in initial conditions can lead to big differences in the behaviour of the system.

How does our imperfect knowledge of the current weather lead to poor weather forecasts?