Teacher Notes

Unit Advanced Level, Working with algebraic and graphical techniques

Notes

This is an adaptation of an activity that was written by Sylvia Knight (University of Oxford Atmospheric, Oceanic and Planetary Physics) and Jon Gray (Banbury School) for a Nuffield project linked to the climate*prediction*.net project.

This version, Climate B, of the activity assumes students have use of a graphic calculator, but there is also an alternative version, Climate A, that uses a spreadsheet instead.
Either of the activities can be used independently, or students can use both – this would allow them to compare the methods used (useful for the Applying Mathematics part of the course).
The work may be shared between students if you wish.

The activity can also be adapted for use as an assignment, but in this case it is recommended that students are given less help than is currently written into the activity to enable them to achieve high marks for their Coursework Portfolio by working independently. It is also advised that a different percentage increase and decrease are used to avoid the possibility of students using the answers given in the Teacher Notes. Values of incoming solar radiation of between 20% (80% decrease) and 200% (100% increase) could be used.

Prior knowledge needed

Before using this activity students will need to have learnt how to use a graphic calculator to:

- draw graphs (including exponential functions);
- enter data and find polynomial regression lines to model the data.

Note that if students also know how to substitute data values into exponential functions and solve the resulting equations, they could be asked to find their own exponential functions to model the temperature data.

Climate*prediction*.net

Some information about the climate*prediction*.net project is given below. Although this activity does not require that students have any detailed knowledge of climate or of the climate*prediction*.net project, it is recommended that they are encouraged to find out more by visiting the climate*prediction*.net website at www.climateprediction.net. Further activities linked to the project and information for teachers is also available from www.climateprediction.net/schools

The climate*prediction*.net project is a joint research project funded by the Natural Environment Research Council (NERC) and the Department of Trade and Industry. Its aim is to use the large number of idle computers worldwide and the power of the internet to predict and understand the climate. Your students can find out more about this project and take part in it by visiting the website at www.climateprediction.net and downloading their own unique simulation model of the Earth's climate. The downloaded program runs as a background process (it does not affect normal computing) to generate data for a climate model. The graphics packages supplied with the model show how weather patterns develop. Results from these experiments will be contributed to the 4th Assessment Report of the Intergovernmental Panel on Climate Change (IPCC).

Answers

5% increase

Time x (years)	Incoming radiation ($Js^{-1}m^{-2}$)	Outgoing radiation ($Js^{-1}m^{-2}$)	Change in temperature (°K)	Temperature of Earth y (°K)
0	364	364	-	283
1	382.2	363.6878571	1.459497345	284.4594973
2	382.2	371.2485893	0.8634092225	285.3229066
3	382.2	375.7764999	0.506428751	285.8293353
4	382.2	378.4515222	0.2955299886	286.1248653
5	382.2	380.0191339	0.1719394816	286.2968048
6	382.2	380.9334091	0.09985802897	286.3966628
7	382.2	381.4651533	0.05793531696	286.4545981
8	382.2	381.773914	0.0335926201	286.4881908

Polynomial functons given by a graphic calculator are listed below:

Quadratic: $y = -0.0869374x^2 + 1.07138397x + 283.282031$

Cubic: $y = 0.01405968x^3 - 0.2556537x^2 + 1.58034468x + 283.045828$

Quartic: $y = -0.001717x^4 + 0.04153642x^3 - 0.3923014x^2 + 1.79427067x + 283.004613$

Graphs can be used to show that the cubic and quartic function given above and the given exponential function, $y = 283 + 3.53\left(1 - e^{-0.54t}\right)$ all give values close to the temperature values for the first 8 years but the values given by the quadratic function are not so good.

In later years only the exponential function gives values that are likely to model what happens in practice i.e. the temperature approaches a new equilibrium value higher than the original 283°K. The quadratic model predicts a temperature fall after 6 years and the quartic model predicts a temperature fall after 7 years, whereas the cubic model predicts a steep rise in temperature after 10 years.

5% decrease

Time (years)	Incoming radiation ($Js^{-1}m^{-2}$)	Outgoing radiation ($Js^{-1}m^{-2}$)	Change in temperature (°K)	Temperature of Earth (°K)
0	364	364	-	283
1	345.8	363.6878571	– 1.410278655	281.5897213
2	345.8	356.4923798	– 0.8429872245	280.7467341
3	345.8	352.2426262	– 0.5079366515	280.2387975
4	345.8	349.700378	– 0.3075058016	279.9312917
5	345.8	348.1679988	– 0.1866930238	279.7445986
6	345.8	347.2401205	– 0.1135391021	279.6310595
7	345.8	346.6767304	– 0.06912142492	279.5619381
8	345.8	346.3340803	– 0.04210688713	279.5198312

Polynomial functons given by a graphic calculator are listed below:

Quadratic: $y = 0.08360494x^2 - 1.0476658x + 282.736059$

Cubic: $y = -0.0131113x^3 + 0.24094152x^2 - 1.5222978x + 282.95633$

Quartic: $y = 0.0016173x^4 - 0.0389895x^3 + 0.36963939x^2 - 1.7237781x + 282.995147$

The cubic and quartic function given above and the given exponential function, $y = 279.45 + 3.55e^{-0.5x}$, all give values close to the temperature values for the first 8 years but the values given by the quadratic function are not so good.

In later years only the exponential function gives values that are likely to model what happens in practice i.e. the temperature approaches a new equilibrium value lower than the original 283°K. The quadratic model predicts a temperature rise after about 6 years and the quartic model predicts a temperature rise after 8 years, whereas the cubic model predicts a steep fall in temperature after 9 years.

Extension

Different percentage changes in the incoming energy give different equilibrium temperatures. If different increments are used (with the same percentage change in the incoming energy) the incremental values of the temperatures are different, but the equilibrium temperature remains the same.